NUMBER AND DENUMERABILITY

(i) No determiner selects for count singular nouns and mass nouns while excluding plural count nouns;
(ii) Some determiners select for plural and mass nouns while excluding singular count nouns (e.g. a lot of, most of).

(1) Cumulativity: the sum of two entities that fall in the denotation of a noun is something that still falls in the denotation of the noun.

Cumulativity groups together plural and mass nouns (boys + boys ⇒ boys; water + water ⇒ water) and excludes singular count nouns (boy + boy ∉ boy).

1. Grammatical number for count nouns

(2) Harry and Ron are wizards.

(3) [[Harry]] ≤, [[Harry and Ron]]

Link (1983): adding structure to the domain $D_e$ of the model via the reflexive, asymmetric and transitive relation $individual-part-of (\leq)$, which creates a mereological structure (a join semi-lattice):

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(5) Atomic individuals: $\text{AT} = \lambda x. (\forall y) (y \leq x \leftrightarrow y = x)$

(an individual is atomic iff it has no individual part except itself)

(6) Sum operator $+$

Given any set $B \subseteq D_e$: $+B = \text{the smallest } x \in D_e \text{ s.t. } \forall b \in B: b \leq_i x$.

(8) Pluralization operator $*$ (= closure under sum) type $<<e,t>, <e,t>>$

Given any set of atomic entities $P$, $*P = \{y \in D_e: \exists X \subseteq P (y = +X)\}$

$[\lambda f_{<<t,e>>}, \lambda x. \exists S \subseteq \{x:f(x)=1\} [x = +S]]$

(9) Hypothesis:

i. Singular count nouns denote a set of atoms.

ii. Pluralization consists in compositionally applying to the denotation of a singular count noun the $*$ operator, yielding a semi-lattice.

(10)

(11) Supremum operator $\sigma$ (type $<<e,t>, e>$): Given any set of entities $P \subseteq A$,

$\sigma x.P(x) := \text{the unique } x \in D_e \text{ such that } x \in P \& \forall y (y \in P \rightarrow y \leq_i x))$

$[\lambda f_{<<t,e>>}, \text{the unique } x \in D_e \text{ such that } [f(x)=1 \& \forall y [f(x)=1 \rightarrow y \leq_i x]]]$}

(12) Hypothesis: The definite determiner denotes the $\sigma$ operator.
When it applies to the denotation of a plural noun, it returns the largest plurality: $\sigma (\{a, b, c, a+b, b+c, a+c, a+b+c\}) = a+b+c$ (maximality effect)

When it applies to the denotation of a singular count noun, $\sigma$ is undefined except when the noun denotes a singleton set (uniqueness condition)

$\sigma (\{a\}) = a$ (because $\leq$ is a reflexive relation)

Does the denotation of a plural noun include the atomic entities? Apparently it doesn’t:

(13) In questa stanza ci sono sedie
    in this room there are chairs

However, (14) is falsified by the presence of even an atomic chair:

(14) In questa stanza non ci sono sedie.
    in this room not there are chairs

(15) Hypothesis: The denotation of a plural noun includes atomic entities, but these can be ‘filtered out’ by a quantity implicature: it is more informative to describe a single chair by using the singular count noun:
Mass nouns inherently denote a (possibly non-atomic) semi-lattice, ordered by the relation $\preceq_m$ (material part of). This solves the following paradox:

(16) This ring is new, but the gold it is made of is ancient.

It is possible to shift the denotation of a mass noun to a count denotation:

(17) a. a lot of beer  (mass)
    b. to have a beer  (count: pre-determined quantity of beer, e.g. a glass)
    c. Irish beers    (count: types of beer)

or vice versa:

(18) a. I ate a chicken.  (count)
    b. I ate chicken.   (mass) (= some unspecified quantity of matter)

NB1: Plural individuals (pluralities) cannot be identified with groups. A group may remain the same even if one or more of its atomic members change (e.g. a committee): a group is an atomic individual of a sort (Barker 1992), which is constituted by ordinary individuals.

→ Linguistic ontology (Link 1983): The individuals in the domain of the model are postulated for a proper account of linguistic phenomena, with no ontological commitment.